

Proximity Effect and Josephson Coupling in the $SO(5)$ Theory of High-Tc Superconductivity

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We consider proximity effect coupling in Superconducting/Antiferromagnetic/Superconducting (S-A-S) sandwiches using the recently developed $SO(5)$ effective theory of high temperature superconductivity. We find that, for narrow junctions, the A region acts like a strong superconductor, and that there is a critical junction thickness which depends on the effective $SO(5)$ coupling constants and on the phase difference across the junction, at which the A region undergoes a Fredericksz-like transition to a state which is intermediate between superconductor and antiferromagnet. For thick junctions, the current-phase relation is sinusoidal, as in standard S-N-S and S-I-S junctions, but for thin junctions it shows a sharp break in slope at the Fredericksz point.

Zhang has recently developed a theory [1] which unifies d-wave superconductivity (S) and antiferromagnetism (A) on the basis of an underlying $SO(5)$ symmetry. The S and A order parameters are combined into a 5-dimensional superspin, and the high energy physics of these superspins is postulated to be rotationally symmetric. At low energies this $SO(5)$ symmetry is broken by a chemical-potential-dependent anisotropy which favors the A state for $\mu < \mu_c$ or the S state for $\mu > \mu_c$. This implies that, at low temperature, there is a “soft direction” for perturbations of a stable d-wave superconductor toward antiferromagnetism. Similarly, the appropriate perturbation, applied to a stable A material, will tend to drive it into the S state. By analogy to the proximity effect in conventional superconductors, it is clear that the relevant perturbing field is provided by proximity of an A material to an S material. Moreover, in a sandwich S-A-S configuration, this proximity effect would be expected to provide a mechanism for Josephson coupling of the two S regions. We also note that one approach to practical high-Tc Josephson junctions involves the use of barriers made from the cuprates near the S/A transition.

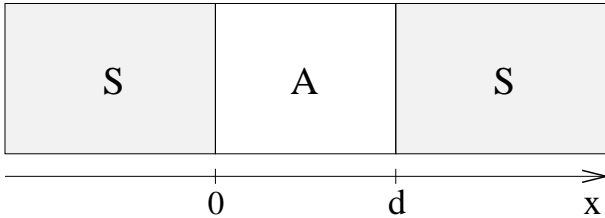


FIG. 1. Geometry of the suggested junction

In this paper we present analytic and numerical results for the properties of the S-A-S Josephson junction system, shown on Fig.1, in terms of $SO(5)$ continuum theory in which the spatial variation of the order parameter is one dimensional. We obtain analytical results for the critical current as a function of thickness and nu-

merical results for the current-phase relation for different thicknesses.

We find that, when the S layers are strongly superconducting, thin A layers are driven completely superconducting by the field of the adjacent S layers, and the $SO(5)$ order parameter lies completely in the superconducting plane. Beyond a critical barrier thickness, we find that the order parameter in the junction starts to tip back toward the antiferromagnetic plane, in a fashion precisely analogous to the Fredericksz transition in liquid crystals. Twisting the superconducting phase, which causes a current to flow through the junction, is analogous to twisting the nematic director at the walls. A sufficiently large twist will drive the system through the Fredericksz transition resulting in a distinctive, non-sinusoidal current-phase relation for an S-A-S junction.

Our results clearly demonstrate that, within $SO(5)$ theory, the details of Josephson coupling through an A barrier are qualitatively different from those of proximity effect junctions with conventional barriers. Hence study of S-A-S junctions provides a critical test of $SO(5)$ theory. By the same token our calculations provide a new basis for the interpretation of real high-Tc Josephson junctions, currently being fabricated and studied [2–4].

In the spirit of $SO(5)$ we describe the system by a three-component order parameter $\mathbf{n} = \{n_x, n_y, n_z\}$, where the first two components are the real and imaginary parts of the superconducting order parameter and the third component represents the antiferromagnetic Neel vector (See Figure 2). For simplicity we treat the Neel vector as a single component. However this component may be viewed as the amplitude of a spatially uniform 3D vector.

According to [1] the system is described by a functional

$$\mathcal{L}(\mathbf{n}) = \frac{\rho}{2} (\partial_\mu n_a)^2 - g n_z^2 \quad (1)$$

with the constraint $n^2 = 1$. As in [5] we assume that the gradient term is $SO(5)$ symmetric. The anisotropy

term g is positive in the A region (so that it would be antiferromagnetic in the absence of proximity effects) and negative in the superconductor.

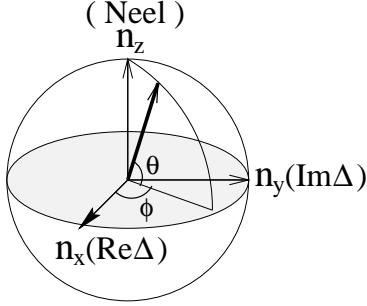


FIG. 2. $SO(5)$ order parameter

The superspin constraint is most naturally implemented in polar coordinates $n_x = \cos \theta \cos \phi$, $n_y = \cos \theta \sin \phi$, and $n_z = \sin \theta$.

$$\mathcal{L}(\theta, \phi) = \frac{\rho}{2} \{ (\partial_\mu \theta)^2 + \cos^2 \theta (\partial_\mu \phi)^2 \} - g \sin^2 \theta \quad (2)$$

In all of our calculations we will assume rigid superconducting boundary conditions $n_z|_0 = 0$, and $n_z|_d = 0$. Strictly speaking this is only true in the case of “strong” superconductors and “weak” antiferromagnets: $|g_S| \gg |g_A|$. However analysis of the general case shows that relaxing this condition does not change the qualitative picture.

At this point one can easily specify the analogy between our problem and the problem of a liquid crystal in a slab with anchoring walls, in an electric field. If the electric field is perpendicular to the walls, it will try to align the director of the liquid crystal along the field. At small voltages the field is unable to overcome the effect of surface pinning, and the equilibrium configuration remains uniform. However with increasing voltage the system will undergo a Fredericksz transition, in which the director begins to align along the field. More interestingly this transition is known to depend on the applied boundary conditions, i.e. on the relative twist of the anchoring directions on the two sides of the slab (the twisted nematic transition) [7].

We now show that similar effects arise in S-A-S sandwiches within $SO(5)$ theory. The role of the voltage is played by $d\sqrt{g_A/\rho}$, and the superconducting phase difference across the junction corresponds to the twist angle imposed by the two anchoring walls. The S-A-S sandwiches will undergo a phase transition in which the A region, between the two superconductors, goes from being purely superconducting (by virtue of the proximity effect) into a mixed S/A state. We also show that, sufficiently close to such a Fredericksz transition, the system possesses non-trivial current-phase characteristics, as a consequence of the transition.

In the A region the Euler-Lagrange equations for the functional (2) are

$$\rho \frac{d^2 \theta}{dx^2} + \rho \cos \theta \sin \theta \left(\frac{d\phi}{dx} \right)^2 + 2g_A \sin \theta \cos \theta = 0 \quad (3)$$

$$\frac{d}{dx} \left(\cos^2 \theta \frac{d\phi}{dx} \right) = 0 \quad (4)$$

The boundary conditions for these equations are given by

$$\theta(x=0) = 0 \quad \phi(x=0) = 0 \quad (5)$$

$$\theta(x=d) = 0 \quad \phi(x=d) = \Delta\Phi \quad (6)$$

where $\Delta\Phi$ is the phase difference between two superconductors.

Equation (4) is nothing but the conservation of current.

$$I_s = n_1 \partial_x n_2 - n_2 \partial_x n_1 = \cos^2 \theta \frac{d\phi}{dx}$$

So we can write (3) as

$$\rho \frac{d^2 \theta}{dx^2} + \rho \sin \theta \frac{I_s^2}{\cos^4 \theta} + 2g_A \sin \theta \cos \theta = 0 \quad (7)$$

The last equation can be easily integrated once giving

$$\xi_A^2 \left(\frac{d\theta}{dx} \right)^2 = -\frac{I_s^2 \xi_A^2}{\cos^2 \theta} - \sin^2 \theta + \frac{I_s^2 \xi_A^2}{\cos^2 \theta_0} + \sin^2 \theta_0 \quad (8)$$

with the characteristic length

$$\xi_A = \sqrt{\rho/2g_A} \quad (9)$$

In writing (8) we expressed the constant of integration in terms of the maximal value θ_0 that will be reached at $x = d/2$ (where $d\theta/dx = 0$). This immediately results in an equation for θ_0

$$\begin{aligned} \frac{d}{2\xi_A} &= \int_0^{\theta_0} \frac{d\theta}{\sqrt{-\frac{\omega_s^2}{\cos^2 \theta} - \sin^2 \theta + \frac{\omega_s^2}{\cos^2 \theta_0} + \sin^2 \theta_0}} = \\ &= \frac{\cos \theta_0}{\sqrt{\omega_s^2 + \cos^2 \theta_0}} K(k) \end{aligned} \quad (10)$$

where $\omega_s = I_s \xi_A$, the parameter k is defined by

$$k^2 = \frac{\sin^2 \theta_0 \cos^2 \theta_0}{\omega_s^2 + \cos^2 \theta_0} \quad (11)$$

and K is the complete elliptic integral of the first kind. Equation (10) should be supplemented by an equation for the current ω_s in terms of the phase difference across the junction

$$\begin{aligned} \Delta\Phi &= 2I_s \int_0^{d/2} \frac{dx}{\cos^2 \theta(x)} = 2I_s \int_0^{\theta_0} \left(\frac{d\theta}{dx} \right)^{-1} \frac{d\theta}{\cos^2 \theta} \\ &= 2\omega_s \int_0^{\theta_0} \frac{d\theta}{\cos^2 \theta \sqrt{-\frac{\omega_s^2}{\cos^2 \theta} - \sin^2 \theta + \frac{\omega_s^2}{\cos^2 \theta_0} + \sin^2 \theta_0}} \\ &= 2\omega_s \frac{\cos \theta_0}{\sqrt{\omega_s^2 + \cos^2 \theta_0}} \Pi_1(-\sin^2 \theta, k) \end{aligned} \quad (12)$$

here $\Pi_1(n, k)$ is a complete elliptic integral of the third kind.

One can easily see that Eq. (10) has a solution only when $d/\xi_A \geq \pi/\sqrt{1+\omega_s^2}$. For smaller d the only solution will be $\theta_0 = 0$, which means that the A region remains uniformly superconducting. Even though antiferromagnetism would be favored in a bulk material of this kind, proximity to a “strong” superconductor forces it to be uniformly superconducting. When $d_c = \pi\xi_A/\sqrt{1+\omega_s^2}$, a second order transition occurs at which θ_0 starts to increase as $\sqrt{d-d_c}$, so that the A region exhibits both kinds of order: superconductivity and antiferromagnetism. It is interesting to note that a non-zero ω_s decreases the critical width of the A region. This can be understood as the result of having an extra “torque” in the x-y plane. This result raises the very interesting possibility of choosing a width of the A region below the critical value at zero current $d_{c0} = \pi\xi$ and then tuning the system through the transition by simply passing a current through the junction!

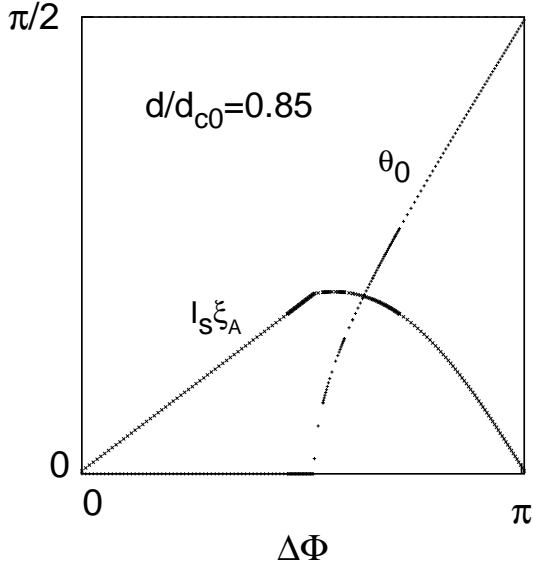


FIG. 3. θ_0 vs $\Delta\phi$ and I_s vs $\Delta\phi$ for an S-A-S junction. The onset of θ_0 and the discontinuity in slope of I_s both occur at the Freedericksz transition.

In Figure 3 we present such an example, for the case $d = 0.85d_{c0}$. This figure shows that the system undergoes a transition when $\Delta\Phi = 1.7$. Below the transition θ_0 is identically zero and I_s is a linear function of $\Delta\Phi$, as one would expect for a uniform superconductor. However above the Freedericksz transition, θ_0 starts to grow and I_s vs $\Delta\Phi$ develops curvature. Eventually, at $\Delta\Phi = \pi$, $\theta_0 = \pi/2$ and I_s goes to zero. We note that further interesting differences with the conventional proximity effect can be expected in the dynamical state at finite voltages. In the presence of a finite voltage across the junction, the full $SO(5)$ order parameter will undergo

periodic motion in $SO(5)$ space, permitting exploration of the low q -vector dynamics of $SO(5)$ theory.

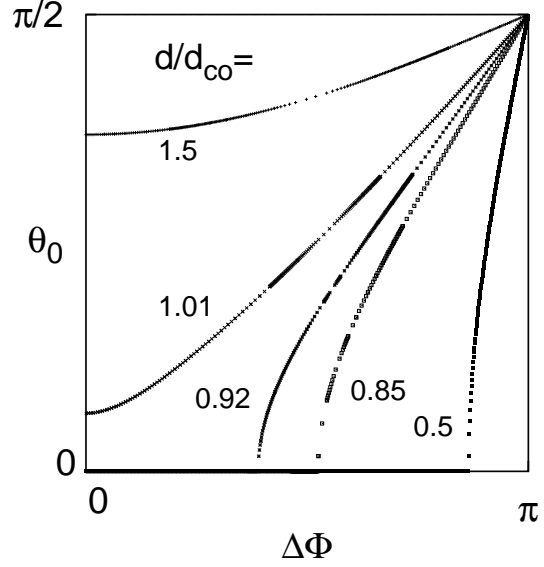


FIG. 4. θ_0 vs $\Delta\Phi$ for junctions with different d/d_{c0} . Notice that for $\Delta\Phi = \pi$ we always have $\theta_0 = \pi/2$.

Figure 4 shows that this feature, $\theta_0 = \pi/2$ when $\Delta\Phi = \pi$, occurs for all widths of the A region. It may be understood as follows: The energy required to twist the superconducting order parameter by π without changing its magnitude is the same as the energy required to rotate the superspin into the antiferromagnetic plane and back into the superconducting plane. However rotating the superspin into the antiferromagnetic direction allows the system to lower its energy because of the g -term.

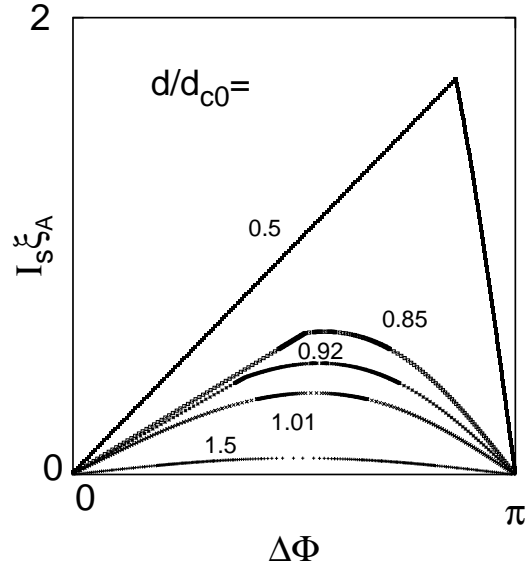


FIG. 5. Current-phase characteristics of junctions with different d/d_{c0} .

This effect is an interesting $SO(5)$ analogue of the result of Krotov *et. al.* [8] that superconductivity between antiferromagnetic stripes is suppressed for nontopological stripes and enhanced for topological stripes.

Figure 5 illustrates the non-trivial current-phase characteristics of S-A-S junctions with increasing width of the A layer. When $d < d_{c0}$ they show a transition from linear dependence below the transition to sin-like dependence above it. Some asymmetry persists in the curves for $d \geq d_{c0}$, and for $d \gg d_{c0}$ they show the usual $\sin(\Delta\Phi)$ dependences of SIS junctions.

It is easy to calculate the critical current of our junctions. For a given d , Eq. (10) does not have any solution for currents that are too large. The first solution appears at a point that corresponds to the maximum of k^2 in Eq. (11) [9]. This k_{max} is given by

$$k_{max}^2 = 1 - 2\omega_s(\sqrt{1 + \omega_s^2} - \omega_s) \simeq 1 - 2\omega_s$$

Using the asymptotic forms of the elliptic functions, we find for Eq. (10) $d/(2\xi_A) = \ln(4/\sqrt{2\omega_s})$ which gives the critical current

$$I_s = \frac{8}{\xi_A} e^{-d/\xi_A}. \quad (13)$$

So ξ_A represents a new correlation length for superconducting proximity effects across antiferromagnets: According to [1] $g_A = 2\chi(\mu_c^2 - \mu^2)$, where μ is the chemical potential, and μ_c is the critical value of the chemical potential at which the first order transition between the superconducting and antiferromagnetic states occurs. In deriving Eq. (9), we have assumed an $SO(5)$ symmetric susceptibility χ for the charge, spin and π operators. According to Eq. (9), when μ is close to μ_c (and hence g_A is small), ξ_A will be large. This provides a new and natural explanation of the long range proximity effect sometimes observed in *PBCO* [2–4]. We note, however, that asymmetry in the χ 's will generate a cut-off for ξ_A in Eq. (9). For $\chi_c > \chi_\pi$ we find $\xi_{max} = \sqrt{\rho/\eta}$ where $\eta = 2\mu_c^2(\chi_c - \chi_\pi)$.

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- [9] This simple argument applies only when w_s is much smaller than one, i.e. well above the Fredericksz transition. For smaller widths one can expect small corrections due to the prefactor of the elliptic function in (10).